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Estimated Zones of Saint-Venant Equations for Flood Routing with Over Bank Unsteady Flow in Open Channel

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In this paper, we learn about the control of open channel water glide under the flood routing conditions. Generally, for flood routing in rivers, the Saint-Venant equations will be used which can be solved by finite distinction method. Saint-Venant equations will be converted into nonlinear equations and will be solved using the Preissmann scheme in the finite difference method. Using the Newton Raphson method, the set of equations will be changed into linear equations and will be solved by the space method. Our aims are to the estimated zones of Saint-Venant equations for flood routing by using the finite difference method with over bank unsteady flow in an open channel. The effectiveness of this method to optimize the choice of finite difference method is more accurate than other methods having adequate space and time steps.

Keywords: Flood routing; Saint-Venant equations; sprace method; over bank flow; hydraulic radius.

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Conflict of interest
The authors declare no conflict of interest in publishing this manuscript in IJIST

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Author's Contribution
All authors have contributed equally

Introduction

Over the last two decades, the study of open channel water flow has become an active research area due to its numerous advantages. It has been extensively investigated by many researchers. The hydraulic and hydrological problems involved in the computation of flood waves are based on Saint Venant (SV) equations, the pair of continuity and momentum equations. This study is important in an open channel under non-uniform or unsteady flow. It is worth mentioning that in this case SV equations cannot be solved analytically. Stoker [1] obtained the approximate solution of SV equations using the explicit finite difference method described in [2]. Ooi and Weyer [3] described the control design for an irrigation channel derived from physical data. Many researchers have obtained approximate solution of SV equations in a particular case of open channels ([4], [5], [7], [8], [9], [10], [11], [12], [13] and [14]). In these models, river waves can be categorize as gravity, diffusion, or kinematic waves, that relates to exceptional types of momentum equations.

The flood routing is fully based on the unsteady flow (long wave or surges) and the water storage equations. At the point of the channel, a flood hydrograph will find out from the well-known hydrograph at various points upstream or downstream via the known channel characters with the characteristics of side inflow or outflow between these two points. Nguyen and Kawano [15] obtained simultaneous solutions for flood routing in the open channel network by using the Preissmann method for their proposed model. Kazezylmaz-Alhan et al. [16] discussed the reliability of the finite difference method for solving proposed diffusion and kinematic wave equations that describe the overland flow. Kohne et al. [17] presented the diffusion and kinematic wave method for runoff and surplus computation. Das [18] presented the Muskingum model to find the flood path and obtained its coefficients by employing the optimization method. Sulistyono and Wiryanto [19] investigated the flood routing by dynamic wave model in the trapezoidal channels.

The main objective of this paper is to develop a quantitative method for identifying river wave types in the case of flood routing with the overbank flow. Different theoretical cases relate to the ratios between the central channel and flooded location concerning breadth and glide will be analyzed. Moreover, we define the estimated zones of SV equations for flood routing by using the Preissmann method with over bank unsteady flow in an open channel.

Materials and Methods

For flood routing issues like river waves and dynamic modeling of one-dimensional, the numerical answer of SV equations will be used. In the case of the flooded region in the river, let B_1 and B_2 be the breath of the central channel and flooded region of the channel, respectively as given in Figure 1. Also let A_1 and A_2 be the cross-sectional area of the central channel and flooded region of the channel, respectively.

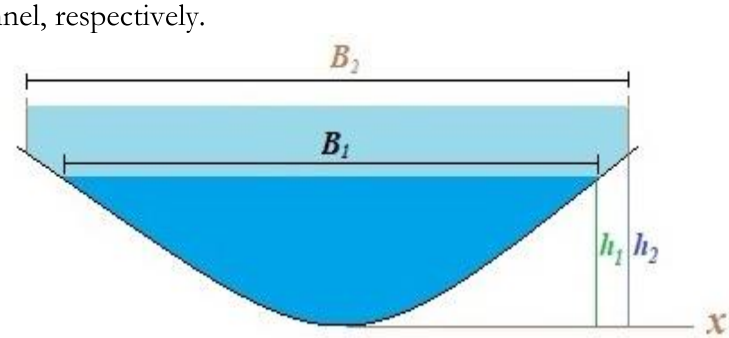

Figure 1. Open Channel having two breaths and cross-sectional areas for flow.

SV equations consist of two equations; continuity equation and momentum equation. The continuity (mass) equation is given as

$$\frac{\partial A_2}{\partial t} + \frac{\partial Q}{\partial x} = q,$$

which further implies that

$$B_2\frac{\partial h}{\partial t}+\frac{\partial Q}{\partial x}\ =q. \qquad (1)$$

Whereas the momentum equation is given as follows:

$$\frac{\partial Q}{\partial t}+\frac{\partial\left(\frac{Q2}{A2}\right)}{\partial x}+\mathrm{g}A_2\ (\frac{\partial h}{\partial x}+S_f)-gA_2S_0+qV=0,$$

$$\frac{\partial VA}{\partial t}+\frac{\partial(V_2A_2)}{\partial x}+\mathrm{gA2}\ (\frac{\partial h}{\partial x}+S_f)-gA_2S_0+qV=0,$$

$$A_2\frac{\partial V}{\partial t}+A_2\frac{\partial(V_2)}{\partial x}+\mathrm{gA2}\ (\frac{\partial h}{\partial x}+S_f)-gA_2S_0+qV=0.$$

Thus, the momentum equation becomes

$$\frac{\partial V}{\partial t}+V\frac{\partial V}{\partial x}+\mathrm{g}\frac{\partial h}{\partial x}+g\left(S_f-S_0\right)+\frac{qV}{A_2}=0, \qquad (2)$$

where is the flow high (m), is the velocity of flow (m/s), is the acceleration due to gravity (m/s^2), is energy line slope, is the slope of the riverbed, is discharge (m^3/s), is the horizontal distance (m), is time (s) and the q is a lateral inflow.

To derive this system, a basic assumption will be made: that there is one-dimensional flow in both regions of the channel (central region and flooded region), and there is lateral inflow or outflow.

Generally, the side channel will be rougher than the central channel. The velocity V in the central region is greater than as compared to the flooded region. In this case, the Manning formula can be applied separately to both regions in determining the velocity of both channels. In this case, the manning formula might be applied one by one to every section in determining the velocity of the section. After that, the discharge in the section will be computed. The total discharge will be equal to the sum of these discharges. Since velocity V in the central channel will be greater than the velocity in the flooded region so the component of discharge in the flooded area is small as compared to the discharge in the central channel; hence discharge Q can be approximated as follows.

$$Q\approx A_1V, \qquad (3)$$

where the cross-sectional area A_1 depends on x and t. As

$$A_1\ =\ B_1h. \qquad (4)$$

Differentiating equation (3), we have

$$\frac{\partial Q}{\partial x}=\ \frac{\partial(A_1V)}{\partial x}\ =\mathrm{V}\frac{\partial A_1}{\partial x}+A_1\frac{\partial V}{\partial x}$$

$$=\mathrm{V}\frac{\partial(B_1h)}{\partial x}+(B_1h)\frac{\partial V}{\partial x}$$

$$=B_1\mathrm{h}\frac{\partial V}{\partial x}+(B_1h)\frac{\partial V}{\partial x}. \qquad (5)$$

Substituting equation (5) in equation (1), we get

$$B_2\frac{\partial h}{\partial t}+(B_1V)\frac{\partial h}{\partial x}+(B_1h)\frac{\partial V}{\partial x}=q,$$

$$B_2\frac{\partial h}{\partial t}+B_1\left(V\frac{\partial h}{\partial x}+h\frac{\partial V}{\partial x}\right)=q$$

and so,

$$\frac{B_2}{B_1}\frac{\partial h}{\partial t}+V\frac{\partial h}{\partial x}+h\frac{\partial V}{\partial x}=\frac{q}{B_1}. \qquad (6)$$

If η is the ratio of flooded region breath B_2 and central channel breath B_2, that is, $\eta=\frac{B_2}{B_1}$, then we obtain

$$\eta\frac{\partial h}{\partial t}+V\frac{\partial h}{\partial x}+\frac{A1}{B1}\frac{\partial V}{\partial x}=\frac{q}{B1}. \qquad (7)$$

Generally, in the momentum equation, the term will be calculated by the Manning formula. Since the velocity in the central region is greater than the velocity in the flooded region, thus the

term of the Manning formula in the flooded region is smaller as compared to the central region of the channel. As

$$Q = \frac{A_1 R^{\frac{2}{3}} S_f^{\frac{1}{2}}}{n},$$

which implies that

$$VA_1 = \frac{A_1 R^{\frac{2}{3}} S_f^{\frac{1}{2}}}{n},$$

and hence

$$S_f = (Vn)^2 R^{-\frac{4}{3}} = (Vn)^2 R^{-m},$$

where R is the hydraulic radius (m), n is the coefficient of roughness and m is constant ($m = \frac{4}{3}$) and ($h << B_1$) is for large rivers. For central channel, the hydraulic radius is given by

$$R = \frac{B_1 h}{B_1 + 2h} = \frac{h}{1 + \frac{2h}{B_1}} \approx h. \tag{9}$$

Substituting equation (9) in equation (8), we get

$$S_f \approx n^2 V^2 h^{-m}. \tag{10}$$

As Froude number is a dimensionless number which is used in hydrodynamics to specify that how a particular model works in relation to a real system, so

$$F = \frac{V}{\sqrt{gh}},$$

that is,

$$gh\, F^2 = V^2. \tag{11}$$

Substituting equation (11) in (10) implies

$$S_f = n^2 g F^2 h^{1-m}.$$

As

$$\frac{\partial S_f}{\partial x} = n^2 g F^2 (1-m) h^{-m} \frac{\partial h}{\partial x},$$

so, we have

$$\frac{\partial h}{\partial x} = \frac{h^m}{n^2 g F^2 (1-m)} \frac{\partial S_f}{\partial x}. \tag{12}$$

Using (12) in (2), the momentum equation becomes

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} + \frac{h^m}{n^2 g F^2 (1-m)} \frac{\partial S_f}{\partial x} + g\left(S_f - S_0\right) + \frac{qV}{A_2} = 0,$$

that is,

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} + \frac{h^m}{\beta}\frac{\partial Sf}{\partial x} + g\left(S_f - S_0\right) + \frac{qV}{A_2} = 0, \tag{13}$$

where $\beta = n^2 F^2 (1-m)$. Continuity equation (7) and the momentum equation (13) provide the general form of SV equations with flooded region and the side channels rougher than the central channel. In such case, SV equations depends on parameter that appears in the continuity equation. In the particular case $\eta = 1$, equivalent to flood routing with no overbank flow, was considered widely by Moussa and Bocquillon [12]. These two equations can be solved by using finite difference method. According to this method, Preissmann method is used to solve SV equations.

Result and Discussion

The Pressman model has a vast application in flood routing hydrograph in the open channel as given in Figure 2.

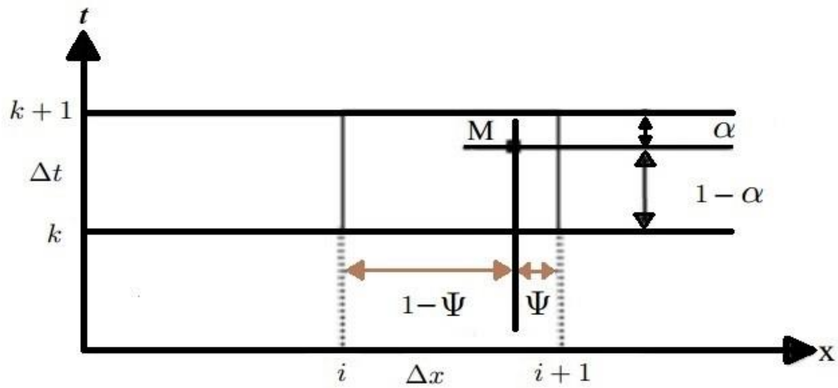

Figure 2. Preissmann method

By using above model, the time derivative is as follows

$$\frac{\partial f}{\partial t}\Big|_i^{k+1} = \psi\left[\frac{f_{i+1}^{k+1}-f_{i+1}^{k}}{\Delta t}\right] + (1-\psi)\left[\frac{f_i^{k+1}-f_i^{k}}{\Delta t}\right]. \tag{14}$$

Similarly, the space derivative is given as follow

$$\frac{\partial f}{\partial x}\Big|_i^{k+1} = \alpha\left[\frac{f_{i+1}^{k+1}-f_i^{k+1}}{\Delta x}\right] + (1-\alpha)\left[\frac{f_{i+1}^{k}-f_i^{k}}{\Delta x}\right]. \tag{15}$$

And the others terms are given as

$$f = \alpha\left[\frac{f_{i+1}^{k+1}+f_i^{k+1}}{2}\right] + (1-\alpha)\left[\frac{f_{i+1}^{k}+f_i^{k}}{2}\right]. \tag{16}$$

In the SV equations; for the terms $\frac{\partial h}{\partial t}$ and $\frac{\partial V}{\partial t}$ we will use equation (14), and for terms $\frac{\partial h}{\partial x}$, $\frac{\partial Sf}{\partial x}$ and $\frac{\partial V}{\partial x}$ we will use equation (15), and for terms $\frac{A_1}{B_1}$, V, $\frac{q}{B_1}$, $\frac{V}{A_2}$ and S_f, we will use equation (16). The continuity equation (7), for $\psi = \frac{1}{2}$ becomes

$$\eta\left[\frac{h_{i+1}^{k+1}-h_{i+1}^{k}}{2\Delta t}+\frac{h_i^{k+1}-h_i^{k}}{2\Delta t}\right] + \left[(\alpha)\frac{V_{i+1}^{k+1}+V_i^{k+1}}{2}+(1-\alpha)\frac{V_{i+1}^{k}+V_i^{k}}{2}\right]\times$$

$$\left[(\alpha)\frac{h_{i+1}^{k+1}-h_i^{k+1}}{\Delta x}+(1-\alpha)\frac{h_{i+1}^{k}-h_i^{k}}{\Delta x}\right] + \left[(\alpha)\frac{\left(\frac{A_1}{B_1}\right)_{i+1}^{k+1}+\left(\frac{A_1}{B_1}\right)_i^{k+1}}{2}\right.$$

$$\left.+(1-\alpha)\frac{\left(\frac{A_1}{B_1}\right)_{i+1}^{k}+\left(\frac{A_1}{B_1}\right)_i^{k}}{2}\right]\times\left[(\alpha)\frac{V_{i+1}^{k+1}+V_i^{k+1}}{\Delta x}+(1-\alpha)\frac{V_{i+1}^{k}+V_i^{k}}{\Delta x}\right]$$

$$-q\left[(\alpha)\frac{\left(\frac{1}{B_1}\right)_{i+1}^{k+1}+\left(\frac{1}{B_1}\right)_i^{k+1}}{2}+(1-\alpha)\frac{\left(\frac{1}{B_1}\right)_{i+1}^{k}+\left(\frac{1}{B_1}\right)_i^{k}}{2}\right]=0,$$

which implies that

$$\eta\left[h_{i+1}^{k+1}+h_i^{k+1}\right]-\eta\left[h_{i+1}^{k}+h_i^{k}\right]+\frac{\Delta t}{\Delta x}\alpha^2\left(V_{i+1}^{k+1}+V_i^{k+1}\right)\left(h_{i+1}^{k+1}-h_i^{k+1}\right)$$
$$+\frac{\Delta t}{\Delta x}\alpha(1-\alpha)\left(V_{i+1}^{k+1}+V_i^{k+1}\right)\left(h_{i+1}^{k}-h_i^{k}\right)$$

$$+\frac{\Delta t}{\Delta x}\alpha(1-\alpha)\left(\mathrm{V}_{\mathbf{i+1}}^{\mathbf{k}}+V_i^k\right)\left(h_{\mathbf{i+1}}^{\mathbf{k+1}}-h_i^{k+1}\right)$$
$$+\frac{\Delta t}{\Delta x}(1-\alpha)^2\left(\mathrm{V}_{\mathbf{i+1}}^{\mathbf{k}}+V_i^k\right)\left(h_{\mathbf{i+1}}^{\mathbf{k}}-h_i^k\right)$$
$$+\frac{\Delta t}{\Delta x}\alpha^2\left(\left(\frac{A_1}{B_1}\right)_{\mathbf{i+1}}^{\mathbf{k+1}}+\left(\frac{A_1}{B_1}\right)_i^{k+1}\right)\left(\mathrm{V}_{\mathbf{i+1}}^{\mathbf{k+1}}-V_i^{k+1}\right)$$
$$+\frac{\Delta t}{\Delta x}\alpha(1-\alpha)\left(\left(\frac{A_1}{B_1}\right)_{\mathbf{i+1}}^{\mathbf{k+1}}+\left(\frac{A_1}{B_1}\right)_i^{k+1}\right)\left(\mathrm{V}_{\mathbf{i+1}}^{\mathbf{k}}-V_i^k\right)$$
$$+\frac{\Delta t}{\Delta x}\alpha(1-\alpha)\left(\left(\frac{A_1}{B_1}\right)_{\mathbf{i+1}}^{\mathbf{k}}+\left(\frac{A_1}{B_1}\right)_i^{k}\right)\left(\mathrm{V}_{\mathbf{i+1}}^{\mathbf{k+1}}-V_i^{k+1}\right)$$
$$+\frac{\Delta t}{\Delta x}(1-\alpha)^2\left(\left(\frac{A_1}{B_1}\right)_{\mathbf{i+1}}^{\mathbf{k}}+\left(\frac{A_1}{B_1}\right)_i^{k}\right)\left(\mathrm{V}_{\mathbf{i+1}}^{\mathbf{k}}-V_i^k\right)-q\Delta t\alpha\left(\left(\frac{1}{B_1}\right)_{\mathbf{i+1}}^{\mathbf{k+1}}+\left(\frac{1}{B_1}\right)_i^{k+1}\right)$$
$$-q\Delta t(1-\alpha)\left(\left(\frac{1}{B_1}\right)_{\mathbf{i+1}}^{\mathbf{k}}+\left(\frac{1}{B_1}\right)_i^{k}\right)=0. \qquad (17)$$

Finally, a function T_i for interval i will be obtained as follows.

$$T_i\left(h_i^{k+1},V_i^{k+1},h_{\mathbf{i+1}}^{\mathbf{k+1}},\mathrm{V}_{\mathbf{i+1}}^{\mathbf{k+1}}\right)=\eta\left[h_{\mathbf{i+1}}^{\mathbf{k+1}}+h_i^{k+1}\right]+\eta D_1$$
$$+\frac{\Delta t}{\Delta x}\alpha^2\left(\left(\frac{A_1}{B_1}\right)_{\mathbf{i+1}}^{\mathbf{k+1}}+\left(\frac{A_1}{B_1}\right)_i^{k+1}\right)\left(\mathrm{V}_{\mathbf{i+1}}^{\mathbf{k+1}}-V_i^{k+1}\right)+\frac{\Delta t}{\Delta x}\alpha D_2\left(\mathrm{V}_{\mathbf{i+1}}^{\mathbf{k+1}}-V_i^{k+1}\right)$$
$$+\frac{\Delta t}{\Delta x}\alpha D_3\left(\left(\frac{A_1}{B_1}\right)_{\mathbf{i+1}}^{\mathbf{k+1}}+\left(\frac{A_1}{B_1}\right)_i^{k+1}\right)+D_4+\frac{\Delta t}{\Delta x}\alpha^2\left(\mathrm{V}_{\mathbf{i+1}}^{\mathbf{k+1}}+V_i^{k+1}\right)\left(\mathrm{h}_{\mathbf{i+1}}^{\mathbf{k+1}}-h_i^{k+1}\right)$$
$$+\frac{\Delta t}{\Delta x}\alpha D_5\left(\mathrm{h}_{\mathbf{i+1}}^{\mathbf{k+1}}-h_i^{k+1}\right)+\frac{\Delta t}{\Delta x}\alpha D_6\left(\mathrm{V}_{\mathbf{i+1}}^{\mathbf{k+1}}+V_i^{k+1}\right)+D_7-D_8$$
$$-q\Delta t\alpha\left(\left(\frac{1}{B_1}\right)_{\mathbf{i+1}}^{\mathbf{k+1}}+\left(\frac{1}{B_1}\right)_i^{k+1}\right)=0. \qquad (18)$$

In equation (18), D_1 to D_8 are coefficients which are given below:

$$D_1=-\mathrm{h}_{\mathbf{i+1}}^{\mathbf{k}}-h_i^k,$$
$$D_2=(1-\alpha)\left(\left(\frac{A_1}{B_1}\right)_{\mathbf{i+1}}^{\mathbf{k}}+\left(\frac{A_1}{B_1}\right)_i^{k}\right),$$
$$D_3=(1-\alpha)\left(\mathrm{V}_{\mathbf{i+1}}^{\mathbf{k}}-V_i^k\right),$$
$$D_4=\frac{\Delta t}{\Delta x}D_2D_3,$$
$$D_5=\alpha(1-\alpha)\left(\mathrm{V}_{\mathbf{i+1}}^{\mathbf{k}}+V_i^k\right),$$
$$D_6=(1-\alpha)\left(\mathrm{h}_{\mathbf{i+1}}^{\mathbf{k}}-h_i^k\right),$$
$$D_7=\frac{\Delta t}{\Delta x}D_5D_6,$$
$$D_8=q\Delta t(1-\alpha)\left(\left(\frac{1}{B_1}\right)_{\mathbf{i+1}}^{\mathbf{k}}+\left(\frac{1}{B_1}\right)_i^{k}\right).$$

Similarly, the momentum equation (13), for $\psi=\frac{1}{2}$ becomes

$$\left[\frac{\mathrm{V}_{\mathbf{i+1}}^{\mathbf{k+1}}-V_{i+1}^k}{2\Delta \mathrm{t}}+\frac{V_i^{k+1}-V_i^k}{2\Delta \mathrm{t}}\right]+\left[(\alpha)\frac{\mathrm{V}_{\mathbf{i+1}}^{\mathbf{k+1}}+V_i^{k+1}}{2}+(1-\alpha)\frac{V_{i+1}^k+V_i^k}{2}\right]$$
$$\times\left[(\alpha)\left(\frac{\mathrm{V}_{\mathbf{i+1}}^{\mathbf{k+1}}-V_i^{k+1}}{2}\right)+(1-\alpha)\left(\frac{V_{i+1}^k-V_i^k}{2}\right)\right]$$

$$+\frac{1}{\beta}\left[(\alpha)\left(\frac{(h^m)_{\mathbf{i+1}}^{\mathbf{k+1}}+(h^m)_i^{k+1}}{2}\right)+(1-\alpha)\left(\frac{(h^m)_{i+1}^{k}-(h^m)_i^{k}}{2}\right)\right]$$

$$\times\left[(\alpha)\left(\frac{(S_f)_{\mathbf{i+1}}^{\mathbf{k+1}}-(S_f)_i^{k+1}}{\Delta \mathrm{x}}\right)+(1-\alpha)\left(\frac{(S_f)_{i+1}^{k}-(S_f)_i^{k}}{\Delta \mathrm{x}}\right)\right]$$

$$-gS_0+g\left[(\alpha)\left(\frac{(S_f)_{\mathbf{i+1}}^{\mathbf{k+1}}+(S_f)_i^{k+1}}{2}\right)+(1-\alpha)\left(\frac{(S_f)_{i+1}^{k}+(S_f)_i^{k}}{2}\right)\right]$$

$$+q\left[\alpha\frac{\left(\frac{V}{A_2}\right)_{\mathbf{i+1}}^{\mathbf{k+1}}+\left(\frac{V}{A_2}\right)_i^{k+1}}{2}+(1-\alpha)\frac{\left(\frac{V}{A_2}\right)_{i+1}^{k}+\left(\frac{V}{A_2}\right)_i^{k}}{2}\right]=0,$$

which implies that

$$\frac{\Delta x}{g\Delta t}\left[\mathrm{V}_{\mathbf{i+1}}^{\mathbf{k+1}}+V_i^{k+1}\right]-\frac{\Delta x}{g\Delta t}\left[\mathrm{V}_{\mathbf{i+1}}^{\mathbf{k}}+V_i^{k}\right]+\frac{1}{g}\alpha^2\left(\mathrm{V}_{\mathbf{i+1}}^{\mathbf{k+1}}+V_i^{k+1}\right)\left(\mathrm{V}_{\mathbf{i+1}}^{\mathbf{k+1}}-V_i^{k+1}\right)$$
$$+\frac{1}{g}\alpha(1-\alpha)\left(\mathrm{V}_{\mathbf{i+1}}^{\mathbf{k}}+V_i^{k}\right)\left(\mathrm{V}_{\mathbf{i+1}}^{\mathbf{k+1}}-V_i^{k+1}\right)+\frac{1}{g}(1-\alpha)^2\left(\mathrm{V}_{\mathbf{i+1}}^{\mathbf{k}}+V_i^{k}\right)\left(\mathrm{V}_{\mathbf{i+1}}^{\mathbf{k}}-V_i^{k}\right)$$
$$+\frac{1}{g}\alpha(1-\alpha)\left(\mathrm{V}_{\mathbf{i+1}}^{\mathbf{k}}+V_i^{k}\right)\left(\mathrm{V}_{\mathbf{i+1}}^{\mathbf{k+1}}-V_i^{k+1}\right)+\frac{1}{g\beta}\alpha^2\left((h^m)_{\mathbf{i+1}}^{\mathbf{k+1}}+(h^m)_i^{k+1}\right)$$
$$\times\left((S_f)_{\mathbf{i+1}}^{\mathbf{k+1}}-(S_f)_i^{k+1}\right)+\frac{1}{g\beta}\alpha(1-\alpha)\left((h^m)_{\mathbf{i+1}}^{\mathbf{k+1}}+(h^m)_i^{k+1}\right)\left((S_f)_{\mathbf{i+1}}^{\mathbf{k}}-(S_f)_i^{k}\right)$$
$$+\frac{1}{g\beta}\alpha(1-\alpha)\left((h^m)_{\mathbf{i+1}}^{\mathbf{k}}+(h^m)_i^{k}\right)\left((S_f)_{\mathbf{i+1}}^{\mathbf{k+1}}-(S_f)_i^{k+1}\right)+\frac{1}{g\beta}(1-\alpha)^2$$
$$\times\left((h^m)_{\mathbf{i+1}}^{\mathbf{k}}+(h^m)_i^{k}\right)\left((S_f)_{\mathbf{i+1}}^{\mathbf{k}}-(S_f)_i^{k}\right)-\frac{2\Delta x}{g}gS_0+\Delta x\,\alpha\left((S_f)_{\mathbf{i+1}}^{\mathbf{k+1}}+(S_f)_i^{k+1}\right)$$
$$+\Delta x(1-\alpha)\left((S_f)_{\mathbf{i+1}}^{\mathbf{k}}+(S_f)_i^{k}\right)+\frac{q\Delta x}{g}\alpha\left(\left(\frac{V}{A_2}\right)_{\mathbf{i+1}}^{\mathbf{k+1}}+\left(\frac{V}{A_2}\right)_i^{k+1}\right)$$
$$+\frac{q\Delta x}{g}(1-\alpha)\left(\left(\frac{V}{A_2}\right)_{\mathbf{i+1}}^{\mathbf{k}}+\left(\frac{V}{A_2}\right)_i^{k}\right)=0. \tag{19}$$

Consequently, a function U_i for interval i will be obtained as follows.

$$U_i\left(h_i^{k+1},V_i^{k+1},h_{\mathbf{i+1}}^{\mathbf{k+1}},\mathrm{V}_{\mathbf{i+1}}^{\mathbf{k+1}}\right)=\frac{\Delta x}{g\Delta t}\left[\mathrm{V}_{\mathbf{i+1}}^{\mathbf{k+1}}+V_i^{k+1}\right]+D_9$$
$$+\frac{\alpha^2}{g}\left(\left(\mathrm{V}_{\mathbf{i+1}}^{\mathbf{k+1}}\right)^{\mathbf{2}}+\left(V_i^{k+1^2}\right)\right)+D_{10}\left(\mathrm{V}_{\mathbf{i+1}}^{\mathbf{k+1}}-V_i^{k+1}\right)+D_{11}\left(\mathrm{V}_{\mathbf{i+1}}^{\mathbf{k+1}}+V_i^{k+1}\right)+D_{12}$$
$$+\frac{1}{g\beta}\alpha^2\left((h^m)_{\mathbf{i+1}}^{\mathbf{k+1}}+(h^m)_i^{k+1}\right)\left((S_f)_{\mathbf{i+1}}^{\mathbf{k+1}}-(S_f)_i^{k+1}\right)+\alpha D_{13}\left((h^m)_{\mathbf{i+1}}^{\mathbf{k+1}}+(h^m)_i^{k+1}\right)$$
$$+\frac{1}{g\beta}\alpha D_{14}\left((S_f)_{\mathbf{i+1}}^{\mathbf{k+1}}-(S_f)_i^{k+1}\right)+D_{15}+D_{16}+\Delta x\,\alpha\left((S_f)_{\mathbf{i+1}}^{\mathbf{k+1}}+(S_f)_i^{k+1}\right)+D_{17}$$
$$+\frac{q\Delta x}{g}\alpha\left(\left(\frac{V}{A_2}\right)_{\mathbf{i+1}}^{\mathbf{k+1}}+\left(\frac{V}{A_2}\right)_i^{k+1}\right)+D_{18}=0, \tag{20}$$

where D_9 to D_{18} are given as:

$$D_9=-\frac{\Delta x}{g\Delta t}\left[\mathrm{V}_{\mathbf{i+1}}^{\mathbf{k}}+V_i^{k}\right],$$

$$D_{10} = \frac{1}{g}\,\alpha(1-\alpha)\left(\mathrm{V}_{\mathbf{i+1}}^{\mathbf{k}} + V_i^k\right),$$
$$D_{11} = \frac{1}{g}\,\alpha(1-\alpha)\left(\mathrm{V}_{\mathbf{i+1}}^{\mathbf{k}} - V_i^k\right),$$
$$D_{12} = \frac{1}{g}\alpha^2\left[\left(V_{i+1}^k\right)^{\mathbf{2}} - \left(V_i^k\right)^2\right],$$
$$D_{13} = \frac{1}{g\beta}(1-\alpha)\,\left(\left(S_f\right)_{\mathbf{i+1}}^{\mathbf{k}} - \left(S_f\right)_i^k\right),$$
$$D_{14} = (1-\alpha)\left((h^m)_{\mathbf{i+1}}^{\mathbf{k}} + (h^m)_i^k\right),$$
$$D_{15} = D_{14}D_{13},$$
$$D_{16} = -2\Delta x S_0,$$
$$D_{17} = \Delta x(1-\alpha)\left(\left(S_f\right)_{\mathbf{i+1}}^{\mathbf{k}} + \left(S_f\right)_i^k\right),$$
$$D_{18} = \frac{q\Delta x}{g}(1-\alpha)\left(\left(\frac{V}{A_2}\right)_{\mathbf{i+1}}^{\mathbf{k}} + \left(\frac{V}{A_2}\right)_i^k\right).$$

As in favor of each interval, the two vector functions and are the functions of four variables, namely. In the favor of each interval, we define two equations, and similarly, for intervals equation will arise. For each interval, we have a node. And for each node, there will be two unknowns (flow rate and depth), thus if there are unknowns, then two equations are formed. Thus, we will obtain two more equations from up and downstream conditions [20].

Finally, a system of nonlinear equations will be generated; to change the nonlinear equations to linear, we will use the Newton-Raphson method. According to this method, the derivative of functions is given as follows

$$\frac{\partial T_i}{\partial \mathrm{h}_{\mathbf{i}}^{\mathbf{k+1}}} = \eta + \frac{\Delta t}{\Delta x}\alpha^2\left[\mathrm{V}_{\mathbf{i+1}}^{\mathbf{k+1}} - V_i^{k+1}\right]\left[\frac{\left[(B_1)_{\mathbf{i}}^{\mathbf{k+1}}\right]^2 - (A_1)_{\mathbf{i}}^{\mathbf{k+1}}\left(\frac{dB_1}{dh}\right)_{\mathbf{i}}^{\mathbf{k+1}}}{\left[(B_1)_{\mathbf{i}}^{\mathbf{k+1}}\right]^2}\right]$$
$$+\frac{\Delta t}{\Delta x}\,\alpha D_3\left[\frac{\left[(B_1)_{\mathbf{i}}^{\mathbf{k+1}}\right]^2 - (A_1)_{\mathbf{i}}^{\mathbf{k+1}}\left(\frac{dB_1}{dh}\right)_{\mathbf{i}}^{\mathbf{k+1}}}{\left[(B_1)_{\mathbf{i}}^{\mathbf{k+1}}\right]^2}\right] - \frac{\Delta t}{\Delta x}\alpha^2\left[\mathrm{V}_{\mathbf{i+1}}^{\mathbf{k+1}} - V_i^{k+1}\right]$$
$$-\frac{\Delta t}{\Delta x}\,\alpha D_5 + q\Delta x\alpha\left[\frac{\left(\frac{dB_1}{dh}\right)_{\mathbf{i}}^{\mathbf{k+1}}}{\left[(B_1)_{\mathbf{i}}^{\mathbf{k+1}}\right]^2}\right],$$

that is,

$$\frac{\partial T_i}{\partial \mathrm{h}_{\mathbf{i}}^{\mathbf{k+1}}} = \eta + \frac{\Delta t}{\Delta x}\alpha^2\left[\mathrm{V}_{\mathbf{i+1}}^{\mathbf{k+1}} - V_i^{k+1}\right]\left[1 - \frac{(A_1)_{\mathbf{i}}^{\mathbf{k+1}}\left(\frac{dB_1}{dh}\right)_{\mathbf{i}}^{\mathbf{k+1}}}{\left[(B_1)_{\mathbf{i}}^{\mathbf{k+1}}\right]^2}\right] + \frac{\Delta t}{\Delta x}\,\alpha D_3$$
$$\left[1 - \frac{(A_1)_{\mathbf{i}}^{\mathbf{k+1}}\left(\frac{dB_1}{dh}\right)_{\mathbf{i}}^{\mathbf{k+1}}}{\left[(B_1)_{\mathbf{i}}^{\mathbf{k+1}}\right]^2}\right] - \frac{\Delta t}{\Delta x}\alpha^2\left[\mathrm{V}_{\mathbf{i+1}}^{\mathbf{k+1}} - V_i^{k+1}\right] - \frac{\Delta t}{\Delta x}\,\alpha D_5 + \left[\frac{q\Delta t\alpha}{\left[(B_1)_{\mathbf{i}}^{\mathbf{k+1}}\right]^2}\right]\left(\frac{dB_1}{dh}\right)_{\mathbf{i}}^{\mathbf{k+1}}. \tag{21}$$

Also

$$\frac{\partial T_i}{\partial \mathrm{h}_{\mathrm{i+1}}^{\mathrm{k+1}}} = \eta + \frac{\Delta t}{\Delta x}\alpha^2[\mathrm{V}_{\mathrm{i+1}}^{\mathrm{k+1}} - V_i^{k+1}]\left[\frac{(B_1)_{\mathrm{i+1}}^{\mathrm{k+1}}\left(\frac{dA_1}{dh}\right)_{\mathrm{i}}^{\mathrm{k+1}} - (A_1)_{\mathrm{i+1}}^{\mathrm{k+1}}\left(\frac{dB_1}{dh}\right)_{\mathrm{i+1}}^{\mathrm{k+1}}}{\left[(B_1)_{\mathrm{i+1}}^{\mathrm{k+1}}\right]^2}\right]$$
$$+\frac{\Delta t}{\Delta x}\alpha D_3\left[\frac{(B_1)_{\mathrm{i+1}}^{\mathrm{k+1}}\left(\frac{dA_1}{dh}\right)_{\mathrm{i}}^{\mathrm{k+1}} - (A_1)_{\mathrm{i+1}}^{\mathrm{k+1}}\left(\frac{dB_1}{dh}\right)_{\mathrm{i+1}}^{\mathrm{k+1}}}{\left[(B_1)_{\mathrm{i+1}}^{\mathrm{k+1}}\right]^2}\right] + \frac{\Delta t}{\Delta x}\alpha^2[\mathrm{V}_{\mathrm{i+1}}^{\mathrm{k+1}} - V_i^{k+1}]$$
$$+\frac{\Delta t}{\Delta x}\alpha D_5 + q\Delta x\alpha\frac{\left(\frac{dB_1}{dh}\right)_{\mathrm{i}}^{\mathrm{k+1}}}{\left[(B_1)_{\mathrm{i}}^{\mathrm{k+1}}\right]^2},$$

that is,

$$\frac{\partial T_i}{\partial \mathrm{h}_{\mathrm{i+1}}^{\mathrm{k+1}}} = \eta + \frac{\Delta t}{\Delta x}\alpha^2[\mathrm{V}_{\mathrm{i+1}}^{\mathrm{k+1}} - V_i^{k+1}]\left[1 - \frac{(A_1)_{\mathrm{i+1}}^{\mathrm{k+1}}\left(\frac{dB_1}{dh}\right)_{\mathrm{i+1}}^{\mathrm{k+1}}}{\left[(B_1)_{\mathrm{i+1}}^{\mathrm{k+1}}\right]^2}\right]$$
$$+\frac{\Delta t}{\Delta x}\alpha D_3\left[1 - \frac{(A_1)_{\mathrm{i+1}}^{\mathrm{k+1}}\left(\frac{dB_1}{dh}\right)_{\mathrm{i+1}}^{\mathrm{k+1}}}{\left[(B_1)_{\mathrm{i+1}}^{\mathrm{k+1}}\right]^2}\right] + \frac{\Delta t}{\Delta x}\alpha^2[\mathrm{V}_{\mathrm{i+1}}^{\mathrm{k+1}} - V_i^{k+1}] + \frac{\Delta t}{\Delta x}\alpha D_5$$
$$+\left[\frac{q\Delta t\alpha}{\left[(B_1)_{\mathrm{i+1}}^{\mathrm{k+1}}\right]^2}\right]\left(\frac{dB_1}{dh}\right)_{\mathrm{i+1}}^{\mathrm{k+1}}. \quad (22)$$

Similarly, we have

$$\frac{\partial T_i}{\partial \mathrm{h}_{\mathrm{i}}^{\mathrm{k+1}}} = -\frac{\Delta t}{\Delta x}\alpha^2\left(\left(\frac{A_1}{B_1}\right)_{\mathrm{i+1}}^{\mathrm{k+1}} + \left(\frac{A_1}{B_1}\right)_i^{k+1}\right) - \frac{\Delta t}{\Delta x}\alpha D_2 + \frac{\Delta t}{\Delta x}\alpha^2[\mathrm{h}_{\mathrm{i+1}}^{\mathrm{k+1}} - h_i^{k+1}]$$
$$+\frac{\Delta t}{\Delta x}\alpha D_6, \quad (23)$$

$$\frac{\partial T_i}{\partial \mathrm{h}_{\mathrm{i+1}}^{\mathrm{k+1}}} = -\frac{\Delta t}{\Delta x}\alpha^2\left(\left(\frac{A_1}{B_1}\right)_{\mathrm{i+1}}^{\mathrm{k+1}} + \left(\frac{A_1}{B_1}\right)_{i+1}^{k+1}\right) - \frac{\Delta t}{\Delta x}\alpha D_2 + \frac{\Delta t}{\Delta x}\alpha^2[\mathrm{h}_{\mathrm{i+1}}^{\mathrm{k+1}} - h_i^{k+1}] + \frac{\Delta t}{\Delta x}\alpha D_6, \quad (24)$$

$$\frac{\partial U_i}{\partial \mathrm{h}_{\mathrm{i}}^{\mathrm{k+1}}} = \frac{\alpha^2}{g\beta}\left((S_f)_{\mathrm{i+1}}^{\mathrm{k+1}} - (S_f)_i^{k+1}\right)m(h^{m-1})_i^{k+1} - \frac{\alpha^2}{g\beta}\left((h^m)_{\mathrm{i+1}}^{\mathrm{k+1}} + (h^m)_i^{k+1}\right)\left(\frac{\partial (S_f)_i^{k+1}}{\partial h_i^{k+1}}\right)$$
$$+\alpha D_{13}\left(m(h^{m-1})_i^{k+1}\right) + \frac{\alpha D_{14}}{g\beta}\frac{\partial (S_f)_i^{k+1}}{\partial h_i^{k+1}} + \Delta x\,\alpha\frac{\partial (S_f)_i^{k+1}}{\partial h_i^{k+1}} - \frac{\mathrm{V}_{\mathrm{i+1}}^{\mathrm{k+1}}\left(\frac{dA_2}{dh}\right)_{\mathrm{i+1}}^{\mathrm{k+1}}}{\left[(A_2)_{\mathrm{i+1}}^{\mathrm{k+1}}\right]^2}. \quad (25)$$

As $S_f = n^2 g F^2 h^{1-m}$, we have

$$\frac{\partial (S_f)_i^{k+1}}{\partial h_i^{k+1}} = n^2 g F^2 (1-m)(h^{-m})_i^{k+1},$$

that is,

$$\frac{\partial (S_f)_i^{k+1}}{\partial h_i^{k+1}} = g\beta(h^{-m})_i^{k+1},$$

where $\beta = n^2F^2(1-m)$, and the above equation (25) becomes

$$\frac{\partial U_i}{\partial h_{\mathrm{i}}^{\mathrm{k+1}}} = \frac{\alpha^2}{g\beta}\left((S_f)_{\mathrm{i+1}}^{\mathrm{k+1}} - (S_f)_i^{k+1}\right)m(h^{m-1})_i^{k+1} - \alpha^2\left((h^m)_{\mathrm{i+1}}^{\mathrm{k+1}} + (h^m)_i^{k+1}\right)(h^{-m})_i^{k+1}$$
$$+\alpha D_{13}\left(m(h^{m-1})_i^{k+1}\right) + \alpha D_{14}(h^{-m})_i^{k+1} + \Delta x\,\alpha g\beta(h^{-m})_i^{k+1} - \frac{\mathrm{V}_{\mathrm{i+1}}^{\mathrm{k+1}}}{\left[(A_2)_{\mathrm{i+1}}^{\mathrm{k+1}}\right]^2}\left(\frac{dA_2}{dh}\right)_{\mathrm{i+1}}^{\mathrm{k+1}}. \quad (26)$$

Also

$$\frac{\partial U_i}{\partial \mathrm{h}_{\mathbf{i+1}}^{\mathbf{k+1}}} = \frac{\alpha^2}{g\beta}\left(\left(S_f\right)_{\mathbf{i+1}}^{\mathbf{k+1}} - \left(S_f\right)_i^{k+1}\right) m\left(h^{m-1}\right)_{i+1}^{k+1} + \frac{\alpha^2}{g\beta}\left(\left(h^m\right)_{\mathbf{i+1}}^{\mathbf{k+1}} + \left(h^m\right)_i^{k+1}\right)\left(\frac{\partial\left(S_f\right)_i^{k+1}}{\partial h_{i+1}^{k+1}}\right)$$

$$+\alpha D_{13}\left(m\left(h^{m-1}\right)_{i+1}^{k+1}\right) + \frac{\alpha D_{14}}{g\beta}\frac{\partial\left(S_f\right)_i^{k+1}}{\partial h_{i+1}^{k+1}} + \Delta x\,\alpha\frac{\partial\left(S_f\right)_i^{k+1}}{\partial h_{i+1}^{k+1}} - \frac{\mathrm{V}_{\mathbf{i+1}}^{\mathbf{k+1}}\left(\frac{dA_2}{dh}\right)_{\mathbf{i+1}}^{\mathbf{k+1}}}{\left[\left(A_2\right)_{\mathbf{i+1}}^{\mathbf{k+1}}\right]^2},$$

that is,

$$\frac{\partial U_i}{\partial \mathrm{h}_{\mathbf{i+1}}^{\mathbf{k+1}}} = \frac{\alpha^2}{g\beta}\left(\left(S_f\right)_{\mathbf{i+1}}^{\mathbf{k+1}} - \left(S_f\right)_i^{k+1}\right) m\left(h^{m-1}\right)_{i+1}^{k+1} + \alpha^2\left(\left(h^m\right)_{\mathbf{i+1}}^{\mathbf{k+1}} + \left(h^m\right)_i^{k+1}\right)\left(h^{-m}\right)_{i+1}^{k+1}$$

$$+\alpha D_{13}\left(m\left(h^{m-1}\right)_{i+1}^{k+1}\right) + \alpha D_{14}\left(h^{-m}\right)_{i+1}^{k+1} + \Delta x\,\alpha g\beta\left(h^{-m}\right)_{i+1}^{k+1} - \frac{\mathrm{V}_{\mathbf{i+1}}^{\mathbf{k+1}}}{\left[\left(A_2\right)_{\mathbf{i+1}}^{\mathbf{k+1}}\right]^2}\left(\frac{dA_2}{dh}\right)_{\mathbf{i+1}}^{\mathbf{k+1}}. \quad (27)$$

In the similar way, the partial derivatives of remaining equations are

$$\frac{\partial U_i}{\partial \mathrm{V}_{\mathbf{i}}^{\mathbf{k+1}}} = \frac{\Delta x}{g\Delta t} - \frac{2\alpha^2}{g}\mathrm{V}_{\mathbf{i}}^{\mathbf{k+1}} - D_{10} + D_{11} + \frac{q\Delta x\alpha}{g\left(A_2\right)_{\mathbf{i}}^{\mathbf{k+1}}} \quad (28)$$

and

$$\frac{\partial U_i}{\partial \mathrm{V}_{\mathbf{i+1}}^{\mathbf{k+1}}} = \frac{\Delta x}{g\Delta t} + \frac{2\alpha^2}{g}\mathrm{V}_{\mathbf{i}}^{\mathbf{k+1}} + D_{10} + D_{11} + \frac{q\Delta x\alpha}{g\left(A_2\right)_{\mathbf{i}}^{\mathbf{k+1}}}. \quad (29)$$

As a result, in support of interval, equations can be produced. Hence, it is ample to get the other two equations from the upstream and downstream boundary conditions, so for all nodes, two unknowns and values will be obtained. The upstream borderline (node No. 1) may additionally be the inflow hydrograph, where is the discharge of influx hydrology, and the downstream boundary circumstance (node No.) may be the glide phase, the place is the overflow height, is the overflow coefficient factor, and is the weight.

Thus, the downstream and upstream conditions are given a

(h_K, V_K) ●———● (h_{K+1}, V_{K+1})

(h_1, V_1) ●———● (h_2, V_2)

As

$$T_0(h_1, V_1) = Q_1 - A_1V_1, \quad (30)$$

$$T_{k+1}(h_{k+1}, V_{k+1}) = A_{k+1}V_{k+1} - CW(h_{k+1} - H)^{1.5}. \quad (31)$$

The initial condition is the steady flow before the flood, so the partial derivative of equation (30) and (31) are

$$\frac{\partial T_0(h_1, V_1)}{\partial h_{\mathbf{1}}} = \frac{\partial Q_1}{\partial h_1} - \frac{\partial A_1V_1}{\partial h_1} = -V_1\frac{\partial A_1}{\partial h_1}, \quad (32)$$

$$\frac{\partial T_0(h_1, V_1)}{\partial V_{\mathbf{1}}} = \frac{\partial Q_1}{\partial V_1} - \frac{\partial A_1V_1}{\partial V_1} = -V_1\frac{\partial V_1}{\partial V_1} = -V_1, \quad (33)$$

$$\frac{\partial T_{k+1}(h_{k+1}, V_{k+1})}{\partial h_{\mathbf{1}}} = \frac{\partial A_{k+1}V_{k+1}}{\partial h_1} - \frac{\partial\left(C.W\left(h_{k+1} - H\right)^{1.5}\right)}{\partial h_1}$$

$$= V_{k+1}\frac{\partial A_{k+1}}{\partial h_1} - (1.5)CW\left(h_{k+1} - H\right)^{0.5}, \quad (34)$$

and

$$\frac{\partial T_{k+1}(h_{k+1}, V_{k+1})}{\partial V_{\mathbf{k+1}}} = \frac{\partial A_{k+1}V_{k+1}}{\partial V_{\mathbf{k+1}}} - \frac{\partial\left(C.W\left(h_{k+1} - H\right)^{1.5}\right)}{\partial V_{\mathbf{k+1}}}$$

$$= A_{k+1} \quad (35)$$

To solve non-linear equations from (32) to (35), we will use the Newton-Raphson algorithm.

The Newton-Raphson method is one of the most familiar iterative schemes used to solve non-linear equations. First, we will write given equations in vector form:

$$T_i(x_1, x_2, \dots, x_{2N}) = 0,$$

where $i = 1,2,3,\dots,2N$, $X = (x_1, x_2, \dots, x_{2N})$denotes the vector of unknown variables. By Taylor series expansion, we have

$$T_i(x+\delta x) = T_i(x) + \sum_{i=1}^{2N} \frac{\partial T_i}{\partial x_j}\delta x_j + O(\delta x)^2$$

for $i = 1,2,3,\dots,2N$. In the above equation, first partial derivatives form a Jacobian matrix. If we take

$$J = \sum_{i=1}^{2N} \frac{\partial T_i}{\partial x_j}$$

then

$$T_i(x+\delta x) = T_i(x) + \mathrm{J}\,\delta x_j + O(\delta x)^2.$$

If we neglect higher order terms and set the left hand-side equal to zero, then we obtain a set of linear equations given by

$$J\,\delta x_j = -T_i(x).$$

The above system of linear equations in matrix form can be further solved by using Gaussian elimination method or LU decomposition method for the unknown value of x, and hence the approximate solution is obtained by

$$x_{new} = x_{old} + \delta x.$$

The iteration procedure will be carried out until a reset convergence stage is achieved. Now in our case, we write our above-mentioned equations in the same way as above

$$T_0(h_1^{n+1}, V_1^{n+1}) = 0.$$

By Taylor series expansion, we can write as follows

$$T_0(h_1^{n+1}, V_1^{n+1}, \Delta h_1^{n+1}, \Delta\, V_1^{n+1}) = T_0(h_1, V_1) + \frac{\partial T_0}{\partial h_1^{n+1}}\Delta h_1^{n+1} + \frac{\partial T_0}{\partial V_1^{n+1}}\Delta V_1^{n+1}.$$

Put $T_0(h_1^{n+1}, V_1^{n+1}, \Delta h_1^{n+1}, \Delta\, V_1^{n+1}) = 0$ and let $T_0(h_1, V_1) = t_0$, so the above equation will become

$$t_0 + \frac{\partial T_0}{\partial h_1^{n+1}}\Delta h_1^{n+1} + \frac{\partial T_0}{\partial V_1^{n+1}}\Delta V_1^{n+1} = 0,$$

that is,

$$\frac{\partial T_0}{\partial h_1^{n+1}}\Delta h_1^{n+1} + \frac{\partial T_0}{\partial V_1^{n+1}}\Delta V_1^{n+1} = -t_0. \tag{36}$$

Also,

$$T_i\left(h_i^{n+1}, V_i^{n+1}\right) = 0,$$

By the Taylor series expansion, we have

$$T_i\left(h_i^{n+1}, V_i^{n+1}, \Delta h_i^{n+1}, \Delta\, V_i^{n+1}\right) = T_i\left(h_i^{n+1}, V_i^{n+1}\right) + \frac{\partial T_i}{\partial h_i^{n+1}}\Delta h_i^{n+1} + \frac{\partial T_i}{\partial V_i^{n+1}}\Delta V_i^{n+1}$$
$$+ \frac{\partial T_i}{\partial h_{i+1}^{n+1}}\Delta h_i^{n+1} + \frac{\partial T_i}{\partial V_{i+1}^{n+1}}\Delta V_i^{n+1}.$$

Put $T_i\left(h_i^{n+1}, V_i^{n+1}, \Delta h_i^{n+1}, \Delta\, V_i^{n+1}\right) = 0$ and $T_i\left(h_i^{n+1}, V_i^{n+1}\right) = t_i$, we get that

$$\frac{\partial T_i}{\partial h_i^{n+1}}\Delta h_i^{n+1} + \frac{\partial T_i}{\partial V_i^{n+1}}\Delta V_i^{n+1} + + \frac{\partial T_i}{\partial h_{i+1}^{n+1}}\Delta h_i^{n+1} + \frac{\partial T_i}{\partial V_{i+1}^{n+1}}\Delta V_i^{n+1} = -t_i. \tag{37}$$

Similarly, we obtain the following equations

$$\frac{\partial U_i}{\partial h_i^{n+1}}\Delta h_i^{n+1} + \frac{\partial U_i}{\partial V_i^{n+1}}\Delta V_i^{n+1} + + \frac{\partial U_i}{\partial h_{i+1}^{n+1}}\Delta h_i^{n+1} + \frac{\partial U_i}{\partial V_{i+1}^{n+1}}\Delta V_i^{n+1} = -u_i. \tag{38}$$

$$\frac{\partial T_{N+1}}{\partial h_{N+1}^{n+1}}\Delta h_1^{n+1} + \frac{\partial T_{N+1}}{\partial V_{N+1}^{n+1}}\Delta V_{N+1}^{n+1} = -t_{N+1}. \tag{39}$$

In the above equation (36) to (39), t_0, t_i, u_i and t_{N+1} be the values of T_0, T_i, U_i and T_{N+1}, respectively. Finally, we get a system of $2N + 2$ equations and $2N + 2$ unknowns, where unknowns are h and V. The system of equations are transformed into the matrix form is as follows

$$\begin{bmatrix} \frac{\partial T_1}{\partial h_1} & \frac{\partial T_1}{\partial V_1} & 0 & . & . & . & . & . & 0 \\ \frac{\partial T_1}{\partial h_1} & \frac{\partial T_1}{\partial V_1} & \frac{\partial T_1}{\partial h_2} & \frac{\partial T_1}{\partial V_2} & . & . & . & . & . \\ \frac{\partial U_1}{\partial h_1} & \frac{\partial U_1}{\partial V_1} & \frac{\partial U_1}{\partial h_2} & \frac{\partial U_1}{\partial V_2} & . & . & . & . & . \\ 0 & 0 & \frac{\partial T_2}{\partial h_2} & \frac{\partial T_2}{\partial V_2} & \frac{\partial T_2}{\partial h_3} & \frac{\partial T_2}{\partial V_3} & . & . & . \\ 0 & 0 & \frac{\partial U_2}{\partial h_2} & \frac{\partial U_2}{\partial V_2} & \frac{\partial U_2}{\partial h_3} & \frac{\partial U_2}{\partial V_3} & . & . & . \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ 0 & . & . & . & . & . & . & \frac{\partial T_{N+1}}{\partial h_{N+1}} & \frac{\partial T_{N+1}}{\partial V_{N+1}} \end{bmatrix} \begin{bmatrix} \Delta h_1 \\ \Delta V_1 \\ \Delta h_2 \\ \Delta V_2 \\ \Delta h_3 \\ \Delta V_3 \\ . \\ . \\ . \\ \Delta h_{N+1} \\ \Delta V_{N+1} \end{bmatrix} = \begin{bmatrix} -t_0 \\ -t_1 \\ -u_1 \\ -t_2 \\ -u_2 \\ . \\ . \\ . \\ . \\ -u_N \\ -t_{N+1} \end{bmatrix}$$

The matrix elimination programs, such as Gaussian elimination or LU decomposition method will be used to solve the above given system. However, Fread [9] used the Sparse characteristics of the Jacobian coefficient matrix with maximum value are considered for consecutive elements and developed an effective algorithm to solve such kind of Ribbon matrix problem. No matter which solution is used to solve the matrix, it is ultimately the case. The step is to calculate the correction value of the unknown of the next iteration. Thus, we obtain the following

$$Q_i^{k+1} = Q_i^k + \delta Q_i,$$
$$h_j^{k+1} = h_j^k + \delta h_j.$$

Numerical Experiment.

The above method requires main and boundary conditions. The main condition is a stable flow before the flow change, where the upstream condition can be flooded hydraulic pressure, and the downstream boundary condition can be a displacement relationship.

We consider a rectangular open channel with a length of $3\ km$, a breath of $3\ m$, having bed slope 0.0005 and a Manning roughness coefficient at the beginning of the channel is 0.25. When the flood entered with the specified water level (in flow water level) with a spillway at its end having height of $1\ m$, width of $15\ m$, and coefficient factor 1.6. Then, the water level curve and flow water level on each point will be drawn at any time by the above method. For calculations, MATLAB software has been used.

In Figures 3 and 4, the flow of water graph at $500\ m$ and $1500\ m$ in intervals from the flood beginning point has been drawn with Preissmann scheme. According to these Figures, we observed that through the increase of distance from the starting point of flood, difference between results increases.

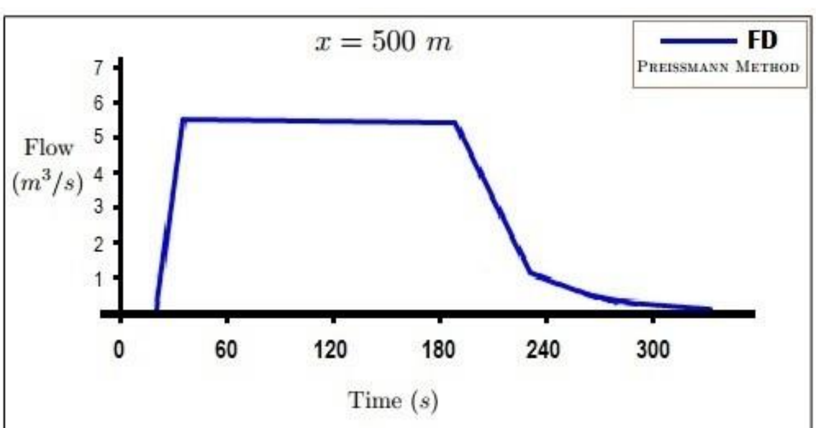

Figure 3. Hydrographs computed at the $500\ m$ distance from the flood start

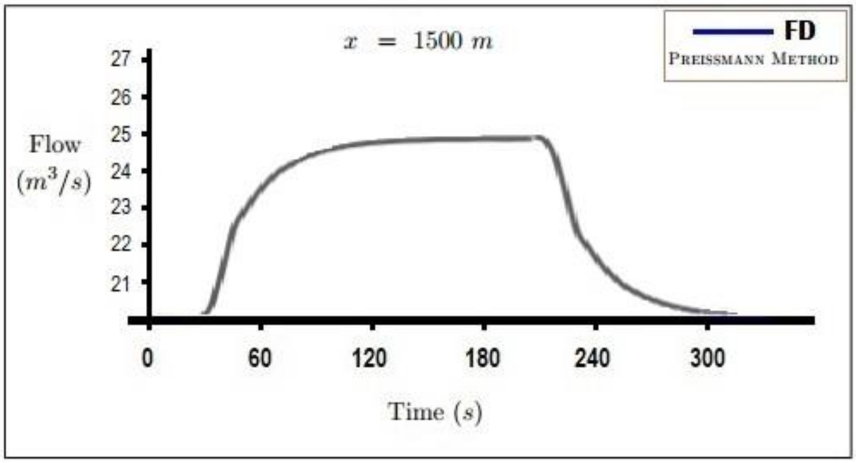

Figure 4. Hydrographs computed at the $1500\ m$ distance from the flood start

Conclusion.

We study the control of open channel water flow under flood routing conditions by employing the Saint Venant equations. The Saint Venant equations converted the structure of flow in the given channel into nonlinear partial differential equations which can be solve numerically by using numerical methods such as finite difference method and Newton Raphson method. By using this method, we develop iteration scheme to estimate the flow and height of water in the given channel. We also study the estimated zone of Saint-Venant equations for flood routing with over bank unsteady flow in the open channel. Some numerical experiments are also presented.

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